Probing Black Hole Microstate Evolution with Networks and Random Walks

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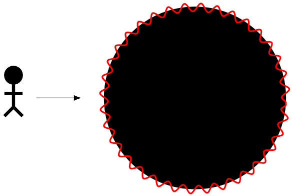
Based on `1812.09328` with Daniel Mayerson (IPhT, CEA Saclay)

Outline

- Introduction
- Network Theory and Quantum Tunneling
- Model 1: N centers
- Model 2: N centers with charge
- Model 3: D1-D5 system (Optional)
- Conclusions

Introduction

- Black hole puzzles and paradoxes in general relativity (singularity, information, …)
- Fuzzball paradigm: black holes are effective geometries, averaged over *microstates*



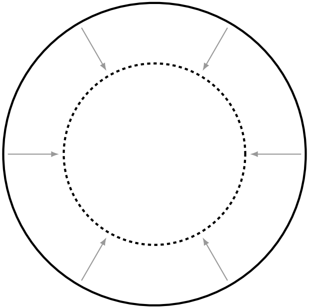
Goal: what do I expect to see?

Same goal, wildly different methods:

- Papadodimas and Raju, [1211.6767], [1310.6334], [1503.08825], …
- Banerjee, Bryan, Papadodimas and Raju, [1603.02812]
- de Boer, van Breukelen, Lokhande, Papadodimas, and Verlinde [1804.10580], [1901.08527]

Black Hole Microstate Formation

Collapsing shell of matter:



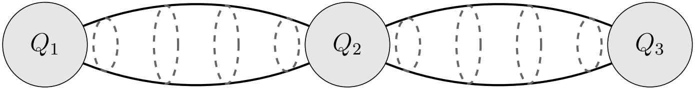
Classical: goes past Schwarzschild radius $\implies$ black hole

Quantum: forms microstate via **tunneling**! (large phase space at horizon [Kraus and Mathur '15])

Black Hole Microstate Formation

Explicit computation: [Bena, Mayerson, Puhm, and Vercnocke ’15]

Smooth, multi-centered, horizonless microstate geometries

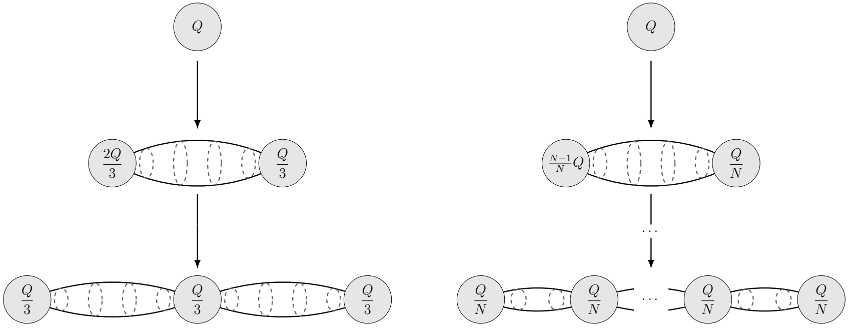


Setup:

- 5D $\mathcal{N} = 1$ SUGRA
- Microstate geometries of three-charge BMPV black holes
- Non-BPS probe branes on BPS backgrounds
- Semi-classical tunneling of charge between centers

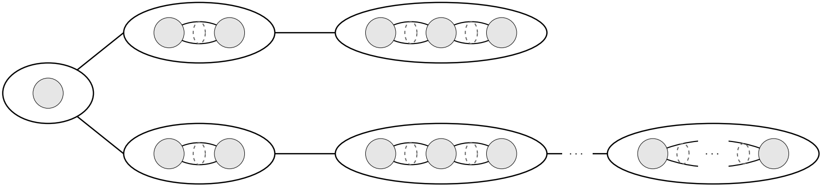
Black Hole Microstate Formation

Tunnel in successive steps:



$\Gamma \sim \exp\left(-N^{-\beta}\right)$ with $\beta > 0 \implies$ easier to form more centers

Time Evolution

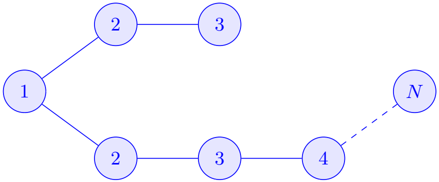


What happens after initial formation? How does the black hole "settle down"?

Many other microstates and paths between microstates!

Time Evolution

Consider more tunneling paths:

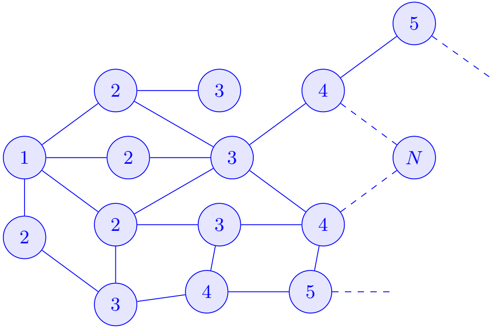


Network - use network theory to understand time evolution!

Works for more general physics questions, too!

Time Evolution

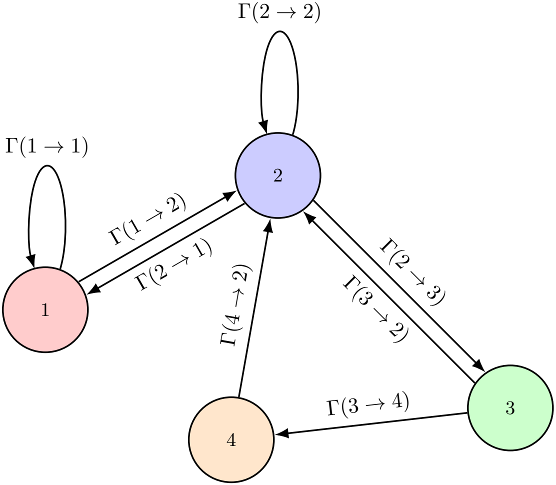
Consider more tunneling paths:



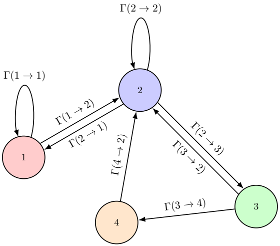
Network - use network theory to understand time evolution!

Works for more general physics questions, too!

Network Theory and Physics



Network Theory and Physics



Physics	Network Theory
State	Node
Transition rate	Edge
Hamiltonian evolution	Stochastic evolution
Late-time probability $\lvert\psi_i\rvert^2$	Eigenvector centrality $p_{\infty,i}$
$\vdots$	$\vdots$

Ex: Ising model, percolation, protein interactions, brain function, quantum cosmology [Hartle and Hertog, '16], string landscape dynamics [Halverson et. al. '17]

Eigenvector Centrality

Transfer matrix $\mathbf{T}$, entries are tunneling probabilities.

Time evolution (discrete) on network:

$$\mathbf{p}(t+1) = \mathbf{p}(t)\mathbf{T}$$

Eigenvector centrality $\mathbf{p}_\infty$: (steady state)

$$\mathbf{p}_\infty = \mathbf{p}_\infty \mathbf{T}$$

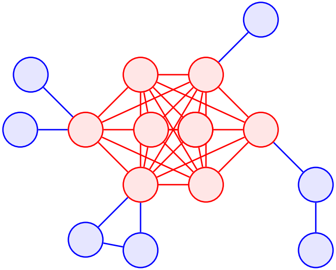
$\mathbf{p}_\infty$ contains probabilities to be at nodes at late times!

Analytic expression for symmetric tunneling: (e.g. ensemble with states at similar energies)

$$p_{\infty,i} = \frac{\text{(sum of edges going out of node } i\text{)}}{\text{(sum of edges going out of } \textbf{all} \text{ nodes)}}$$

Random Walks

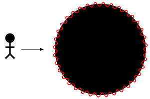
Exact answer needs info about *every node*, hard for large networks



Approximate with a **random walk** on network:

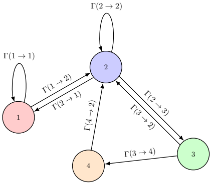
$$p_{\infty,i} \approx \frac{\text{(number steps spent at node } i\text{)}}{\text{(total number of steps)}}$$

Networks and Black Hole Microstates



Goal: what do I expect to see?

In particular: **how many centers**?



$$\rightarrow \mathbf{p}_\infty \rightarrow \langle N \rangle$$

Setting:

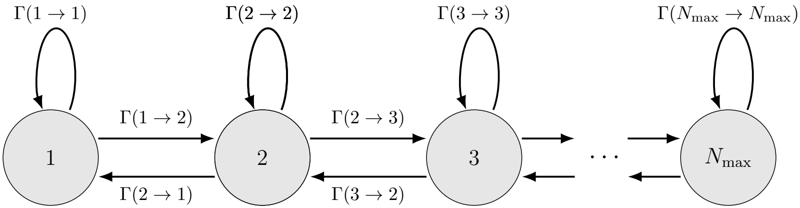
- Smooth, multi-center, horizonless microstate geometries
- Near-BPS (lift off of BPS, what happens?)
- Model known physics

Model 1: N centers

Toy model, very coarse-grained:

- Microstates characterized only by number of centers N
- Finite cut-off, $1 \leq N \leq N_{\text{max}}$
- Tunneling between states with $\Delta N = \pm 1, 0$ (*local* tunneling)

Simple network:



Model 1: N centers

Degeneracy:

$$\omega(N) \sim N^{\beta}$$

- $\beta < 0$: fewer centers $\leftrightarrow$ larger bubbles, more ways to excite
- $\beta > 0$: many centers $\leftrightarrow$ more ways to arrange them

Transition rates:

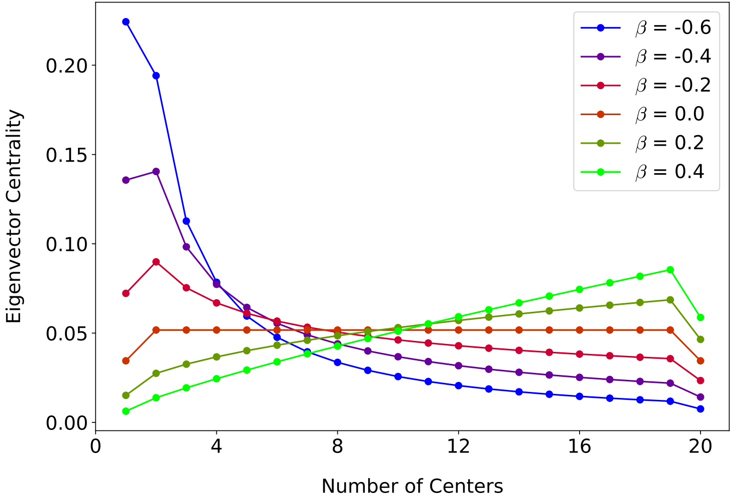
$$\Gamma(N \to N') \sim \exp\left(-\gamma \min(N, N')^{\delta}\right)$$

- $\delta < 0$: many centers $\leftrightarrow$ smaller bubbles, easier to transition to these states
- $\gamma > 0$: exponential suppression (unimportant, set to 1)

Model 1: N centers

$$\omega(N) \sim N^{\beta}\,, \quad \Gamma(N \to N') \sim \text{exp}\left(-\text{min}(N, N')^{\delta}\right)$$

Fix $\delta = 0$, look at β:



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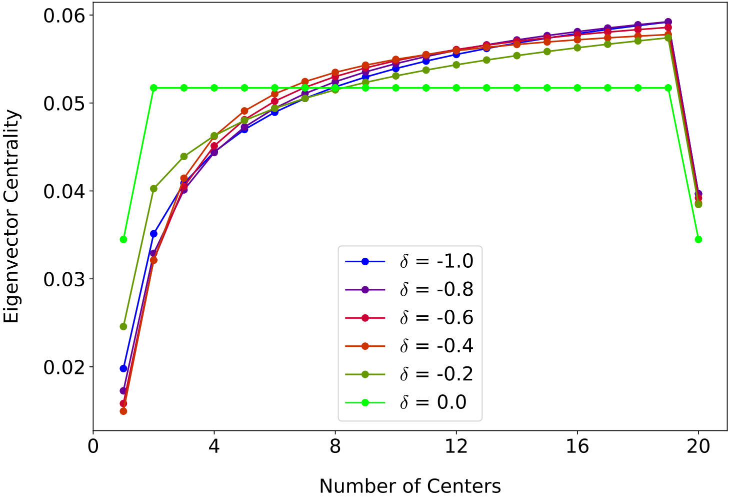
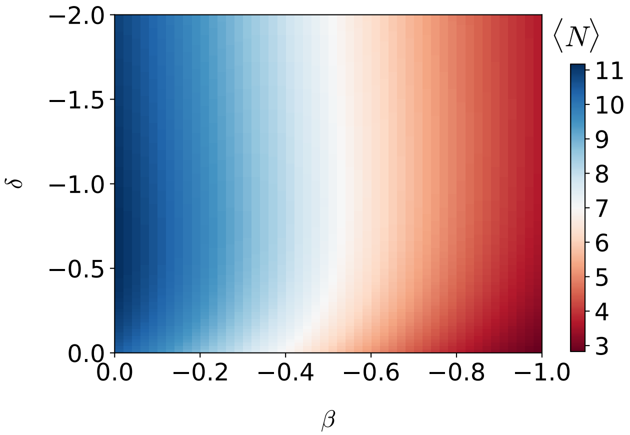


Model 1: N centers

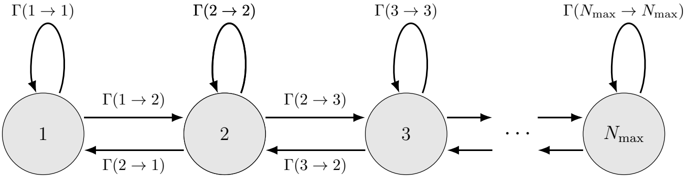
$$\omega(N) \sim N^{\beta}\,, \quad \Gamma(N \to N') \sim \exp\left(-\min(N, N')^{\delta}\right)$$

Study β and δ together:



Effects from β dominate $\implies$ degeneracy more important!

Model 1: N centers



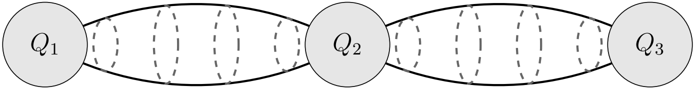
$$\omega(N) \sim N^{\beta}\,, \quad \Gamma(N \to N') \sim \exp\left(-\gamma \min(N, N')^{\delta}\right)$$

Conclusions:

- Degeneracy dominant, insensitive to change in transition rates
- **Ergodic** behavior, all microstates $\approx$ equally likely
- Coarse-grained, not enough features for interesting dynamics

Model 2: N centers with charge

Centers on a line:



Fix total charge Q; microstate with charges $\{Q_i\}$ distributed over N centers.

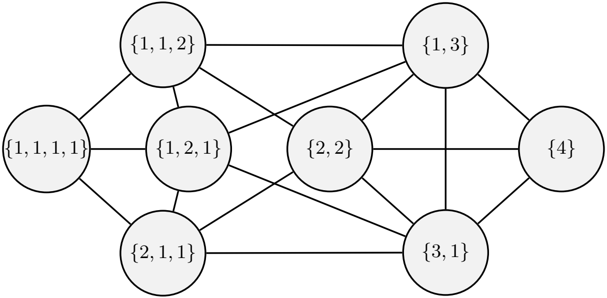
Each center has at least one quanta of charge, $N \leq Q$.

Transitions: charge tunnels *locally*!

- Move from one center to an adjacent center, or
- Split off from one center, form a new adjacent center

Model 2: N centers with charge

Network for microstates with total charge $Q = 4$:



Microstates are compositions of Q, total 2^{Q-1} microstates!

Need to use random walk methods

Model 2: N centers with charge

Degeneracy:

$$\omega(N, \{Q_i\}) \sim \prod_{i=1}^{N} \alpha \, Q_i{}^{\beta}$$

- α: suppress or enhance large number of centers
- $\beta > 0$: fewer centers $\leftrightarrow$ larger bubbles, more ways to excite
- $\beta < 0$: many centers $\leftrightarrow$ more ways to configure, add velocities, etc.

Model 2: N centers with charge

Transition rates: (tunnel q from center n)

$$\Gamma(N, \{Q_i\} \to N', \{Q_i'\}) \sim \exp\left(-\gamma q^{\delta} Q_L^{\lambda} Q_R^{\lambda}\right)$$

- $\delta > 0$: harder to tunnel larger charges
- $Q_L, Q_R =$ amount of charge to left/right of tunneling charge

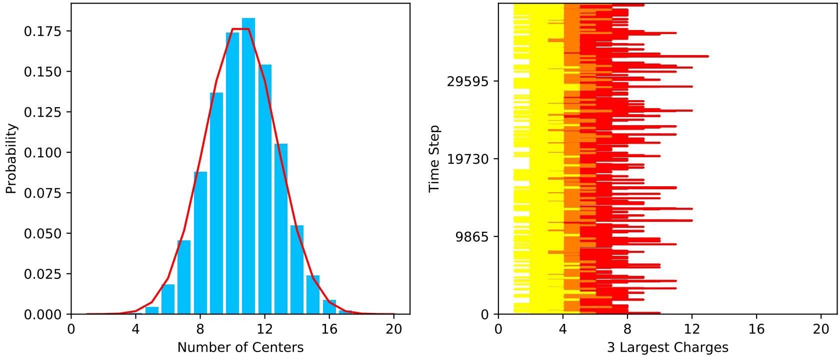
$$Q_L = Q_{n-1} + \omega Q_{n-2} + \omega^2 Q_{n-3} + \dots$$
$$Q_R = Q_{n+1} + \omega Q_{n+2} + \omega^2 Q_{n+3} + \dots$$

- $0 \le \omega \le 1$: attenuation parameter, further centers affect tunneling charge less
- $\lambda > 0$: more charge nearby makes it harder to tunnel

Explicit microstates: more charge nearby $\leftrightarrow$ further distances between centers $\leftrightarrow$ harder to tunnel

Model 2: N centers with charge

Fix parameters, do random walk on network:

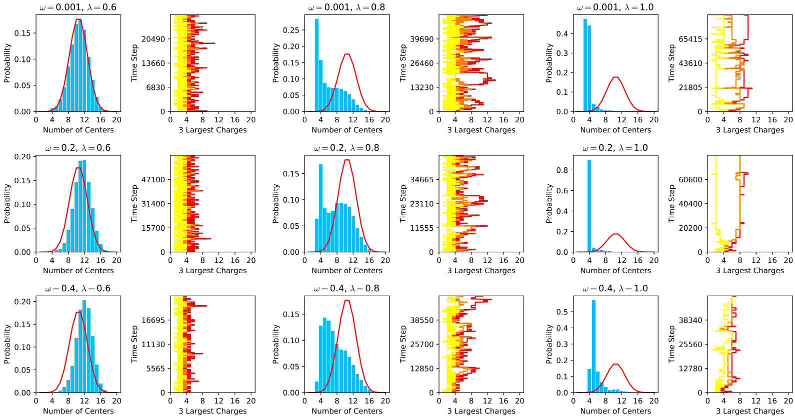


- Red curve: degeneracy
- Blue bars: RW approximation of eigenvector centrality
- Red/orange/yellow: three largest charges (resp.) of microstate at each time step in RW

Model 2: N centers with charge

$$\omega(N, \{Q_i\}) \sim \prod_{i=1}^{N} \alpha {Q_i}^{\beta}\,, \quad \Gamma(N, \{Q_i\} \to N', \{Q_i'\}) \sim \exp\left(-\gamma q^{\delta} Q_L^{\lambda} Q_R^{\lambda}\right)$$

Interesting dependence on λ: (λ increases to the right)

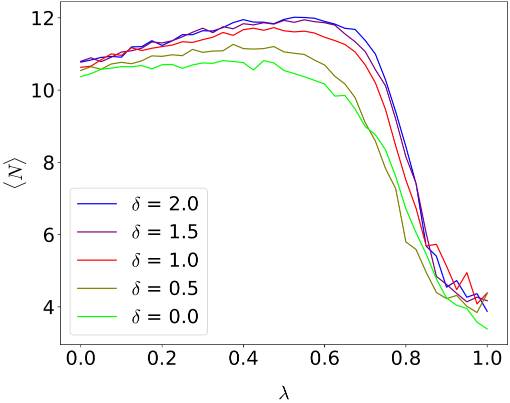


Departure from degeneracy – $\langle N \rangle$ suddenly drops near $\lambda \approx 1$

Model 2: N centers with charge

$$\omega(N,\{Q_i\}) \sim \prod_{i=1}^{N} \alpha Q_i{}^{\beta}\,, \quad \Gamma(N,\{Q_i\} \to N',\{Q_i'\}) \sim \exp\left(-\gamma q^{\delta}\, Q_L^{\lambda}\, Q_R^{\lambda}\right)$$

Investigate this dependence on λ further:

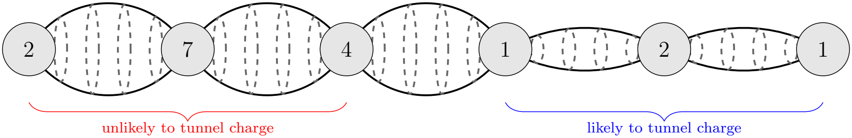


Robust behavior; sudden drop to $\langle N \rangle \approx 3$ near $\lambda = 1$ for *all* values of other parameters! Otherwise, matches degeneracy.

Model 2: N centers with charge

Conclusions:

- Two distinct phases in parameters space:
  - **Ergodic**: degeneracy dominates, all states equally likely, excursions easy
  - **Trapped**: transition rates dominate, system trapped in low N, excursions hard
- Sharp cross-over at $\lambda \approx 1$; true phase transition?
- Entropic arguments *not* sufficient to explain dynamics!



Model 3: D1-D5 system

Setup:

- Type IIB string theory on $\mathbb{R}^{4,1} \times S^1 \times T^4$
- N_1 D1-branes on S^1, N_5 D5-branes on $S^1 \times T^4$
- World-volume sigma model, target space is a deformation of the symmetric product $(T^4)^N/S_N$, with $N = N_1 N_5$
- Ramond ground states dual to Lunin-Mathur supertubes

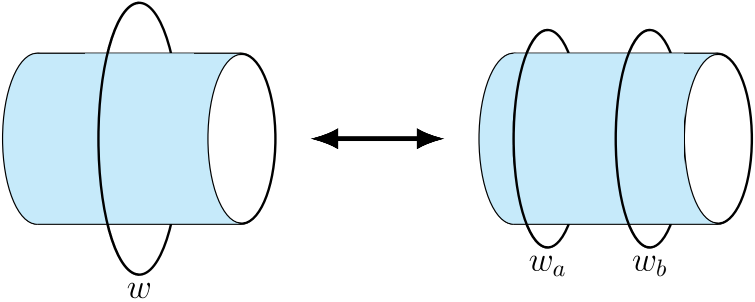
String gas picture:

- Gas of strings with winding numbers $\{w_i\}$ on S^1
- $\sum_i w_i = N$
- $\omega(\{w_i\})$ known exactly; finite number of ways to divide strings into 8 bosonic and 8 fermionic modes

Goal: how many strings in a typical state?

Model 3: D1-D5 system

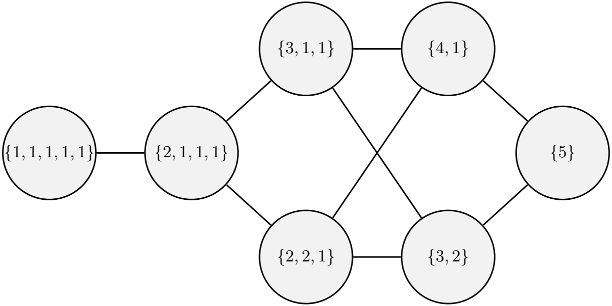
Transitions: splitting and combining adjacent strings



$$\Gamma(\{w_i\} \to \{w_i'\}) = \exp\left(-\gamma w^{\delta} \mathsf{min}(w_a, w_b)^{\lambda_1} \mathsf{max}(w_a, w_b)^{\lambda_2}\right)$$

Model 3: D1-D5 system

Network for $N = N_1 N_5 = 5$:

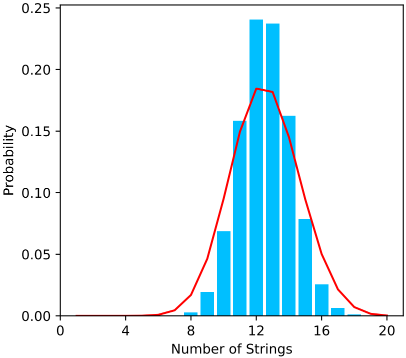


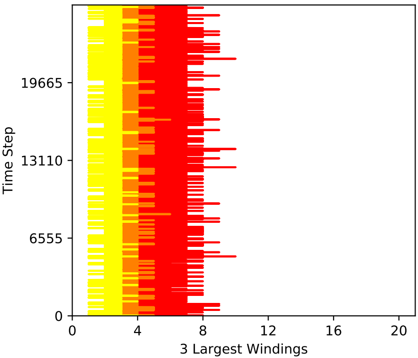
States are partitions of N; number grows exponentially with N!

Once again, need to do random walks!

Model 3: D1-D5 system

Main result: parameters have almost no effect on $\langle N_{strings} \rangle$!





Amplified phase: centrality follows degeneracy, but more sharply peaked

States responsible for most of the entropy are *very highly connected*

Summary of Results

Main takeaway: networks provide useful tools for understanding tunneling in quantum systems, including black hole microstates!

Late-time behavior for three different microstate models:

1. N centers (coarse-grained)
   - **Ergodic** phase: *degeneracy* important
2. N centers with charge (along a line)
   - **Ergodic** phase: *degeneracy* important
   - **Trapped** phase: *transition rates* important, stuck at low N
3. D1-D5 system (string gas picture)
   - **Amplified** phase: *both* important, most degenerate states are highly connected

Interesting dynamics emerge; entropic arguments not sufficient to explain results!

Discussion and Future Directions

- Trapped phase
  - Large $N \leftrightarrow$ large J
  - Microstates in model 2 dynamically shed angular momentum?
  - Similar results found from classical instabilities [Eperon, Reall and Santos '16; Marolf, Michel and Puhm '16]

- Glassy black holes
  - Non-extremal microstates $=$ long-lived metastable states with many different phases ("glassy" physics)
  - Trapped phase $\implies$ random walks stuck in long-lived states
  - Explicit examples of non-ergodic behavior via quiver quantum mechanics [Anninos, Anous, Denef, Konstantinidis and Shaghoulian '12]

Discussion and Future Directions

- Distinguishability of microstates
  - N is not a semi-classical observable, global feature
  - Not doing equilibrium physics; can have deviations from thermal behavior! [Hartle and Hertog, '17]
  - Need to work harder to understand time scales; what does "late-time" mean when we include Hawking radiation?

- Other microstate models: D1-D5-P superstrata, LLM bubbling geometries, fuzzy D-brane geometries in BFSS matrix model, …

- More general Bena-Warner microstates; want enough to account for full entropy of black hole

- More applications of network theory for **dynamical questions**!