Me, you, and $g-2$:

a string theorist's take on the muon anomalous magnetic moment

Disclaimer: I am a string theorist who is often wrong

Motivation

Big question in particle physics: where's the new stuff?

- Hierarchy problem
- Strong CP problem
- Cosmological constant problem
- Dark matter and dark energy
- Baryon asymmetry
- Quantum gravity

A possible window into new physics: a 3σ discrepancy between the theoretical Standard Model value and experimental value of the muon magnetic moment.

The $g-2$ collaboration at Fermilab has been working for years to more precisely measure the muon magnetic moment, and rumor has it they release results this month!

Why stick to particle experiments?

Direct control and generation, sensitive to TeV physics, existing precision calculational tools

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Also particle colliders are rad as hell, let's build new ones!

Goal of this talk: convince a bunch of string theorists that they should care about the $g-2$ experiment.

Another disclaimer: references available upon request

Executive summary

1. Magnetic moments in QFT are simple yet general:

$$\vec{\mu} = g\left(\frac{e}{2m}\right)\vec{S}\,, \quad g = 2(F_1(0) + F_2(0))$$

2. Tension between theory and experiment is significant ($\sim 3\sigma$) for the muon magnetic moment:

$$a_\mu \equiv \frac{g_\mu - 2}{2}\,, \quad \Delta a_\mu(\mathsf{exp} - \mathsf{SM}) = (261 \pm 80) \times 10^{-11}$$

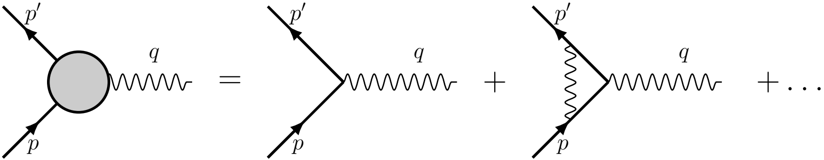
3. Possible stringy resolutions:
   - SUSY ($\checkmark$)
   - Extra dimensions ($\times$)
   - Axions (?)
   - Dark sectors (?)

Magnetic moments in QFT

Consider a spin-1/2 fermion ψ coupled to a $U(1)$ gauge field A_μ.
Want to understand its magnetic moment μ, defined by

$$V = -\vec{\mu} \cdot \vec{B}$$

Three-point $\langle \psi\psi A \rangle$ interaction:



Denote vertex function (gray circle) by $\Gamma^\mu(p', p)$, sum of all internal amputated diagrams.

Scattering amplitude:

$$i\,\mathcal{M} = -ie\,\bar{u}(p')\,\Gamma^\mu(p', p)\,u(p)\,\tilde{A}_\mu(q)$$

Magnetic moments in QFT

Assume Lorentz invariance and parity invariance:

$$\Gamma^\mu(p',p) = A\gamma^\mu + B(p+p')^\mu + C(p-p')^\mu$$

Other terms either violate symmetries or are redundant:

$$\gamma^\mu p_\mu u(p) = mu(p)$$

Functions A, B, and C depend on $p^2 = p'^2 = m^2$ and q^2.

Ward identity:

$$q_\mu \Gamma^\mu = 0 \implies C = 0$$

Left with two independent terms in the vertex function. Can without loss of generality write them as:

$$\Gamma^\mu(p',p) = F_1(q^2)\gamma^\mu + F_2(q^2)\,\frac{i\sigma^{\mu\nu}q_\nu}{2m}\,, \quad \sigma^{\mu\nu} \equiv i\gamma^{[\mu}\gamma^{\nu]}$$

Applies for any spin-1/2 fermion that can exist as an outgoing state; form factors F_1 and F_2 depend on the details.

Magnetic moments in QFT

Specialize to massless, on-shell photons:

$$i\,\mathcal{M} = -ie\,\bar{u}(p')\left[F_1(0)\gamma^\mu + F_2(0)\,\frac{i\sigma^{\mu\nu}q_\nu}{2m}\right]u(p)\,\tilde{A}_\mu(q)$$

Generated by an effective interaction Hamiltonian:

$$H_I = \int d^4x\,\left[e\,F_1(0)\,\bar{\psi}\gamma^\mu A_\mu\psi + \frac{e}{2m}F_2(0)\,\bar{\psi}\sigma^{\mu\nu}F_{\mu\nu}\psi\right]$$

From here, just need to specify the setup:

- Set $A_\mu = (0, \vec{A})$, constant magnetic field $\vec{B}$
- Define a wavepacket $|\psi\rangle$ of ingoing fermions with spin $\vec{S}$, i.e. $|\psi\rangle = \int d^3p\,f(\vec{p})a^\dagger(\vec{p})|0\rangle$.

$$\langle\psi|H_I|\psi\rangle = V = -\vec{\mu}\cdot\vec{B}$$

Magnetic moments in QFT

End result:

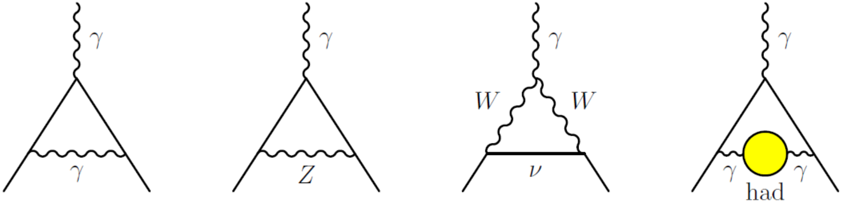
$$\vec{\mu} = g\left(\frac{e}{2m}\right)\vec{S}\,, \quad g = 2\left(F_1(0) + F_2(0)\right)\ .$$

Standard model leptons (electron, muon, tau):

$$F_1(0) = 2 + \text{(loop contributions)} \implies g = 2 + \mathcal{O}(\alpha)$$
$$F_2(0) = 0 + \text{(loop contributions)}$$

Finally, this brings us to the anomalous magnetic moment:

$$\boxed{a = \frac{g-2}{2}}$$



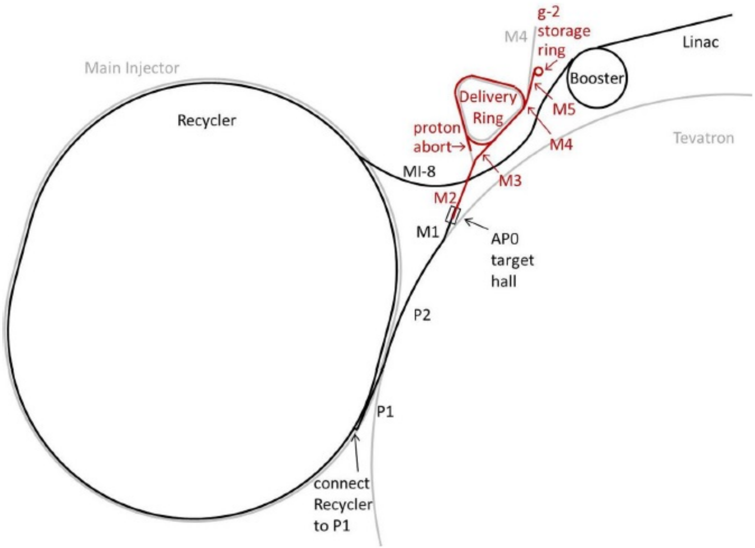
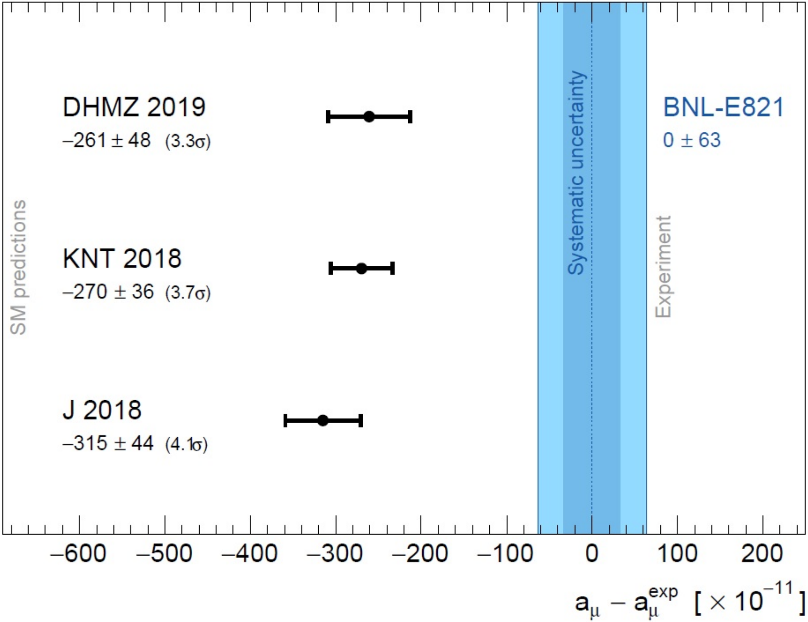


Figure 7.1: Path of the beam to $(g-2)$. Protons (black) are accelerated in the Linac and Booster, are re-bunched in the Recycler, and then travel through the P1, P2, and M1 lines to the AP0 target hall. Secondary beam (red) then travels through the M2 and M3 lines, around the Delivery Ring, and then through the M4 and M5 lines to the muon storage ring.


DHMZ 2019
−261 ± 48 (3.3σ)
KNT 2018
−270 ± 36 (3.7σ)
J 2018
−315 ± 44 (4.1σ)
SM predictions
Systematic uncertainty
BNL-E821
0 ± 63
Experiment
−600 −500 −400 −300 −200 −100 0 100 200
$a_\mu - a_\mu^{exp}$ [$\times 10^{-11}$]

Beyond the Standard Model

If the discrepancy holds up with the new $g - 2$ experiment results, need some BSM physics to explain it!

Non-standard phenomenology!

Flavour and CP preserving. Also, loop-induced, new physics not at tree level

Chirality-flipping interaction: (dimension 5 operator)

$$\mathcal{L} \supset \frac{e}{2m} F_2(0) \bar{\psi}_R \sigma^{\mu\nu} F_{\mu\nu} \psi_L$$

Beyond the Standard Model

Precise gap that needs to be accounted for:

$$\Delta a_\mu\,(\mathsf{exp} - \mathsf{SM}) = (261 \pm 80) \times 10^{-11}$$

EFT estimates for new physics at a cut-off scale Λ:

$$a_\mu(\mathsf{NP}) = \mathcal{O}(1) \times \left(\frac{m_\mu}{\Lambda}\right)^2 \times \left(\frac{\delta m_\mu(\mathsf{NP})}{m_\mu}\right)$$

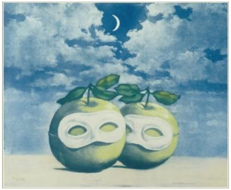
Without fine-tuning, we obtain a great window into accessible physics:

$$\frac{\delta m_\mu(\mathsf{NP})}{m_\mu} \lesssim 1 \implies \Lambda \lesssim 3\ \mathsf{TeV}$$

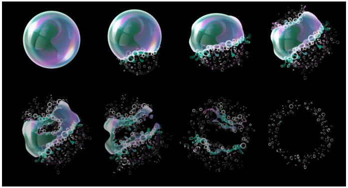
Where are the strings?

No way we're going to "prove" string theory like this, but there are four key aspects that are promising:

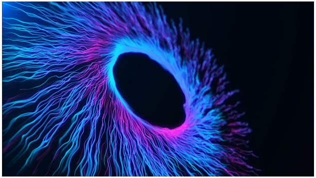
Supersymmetry



Extra dimensions



Axions



Dark sectors

Supersymmetry

Studies consider the Minimal Supersymmetric Standard Model (MSSM): superpartners of SM plus an extra Higgs doublet

$$W = y_u \, \bar{u} Q H_u + y_d \, \bar{d} Q H_d - y_e \, \bar{e} L H_d + \mu H_u H_d$$

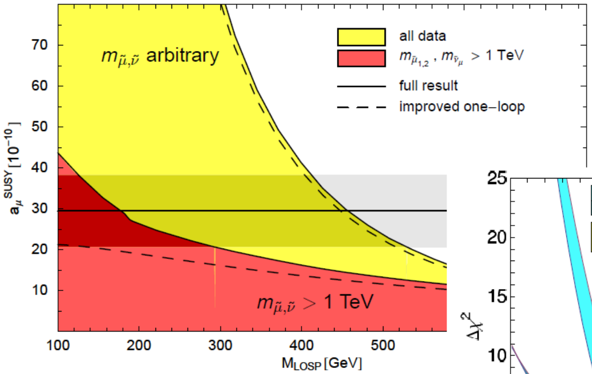
Need Higgs doublet superfields to have opposite hypercharges Y!
(Also important for anomaly cancellation, Tr$[Y^3] = 0$)

$$\tan\beta = \frac{v_2}{v_1} \,, \quad 1 \lesssim \tan\beta \lesssim 50$$

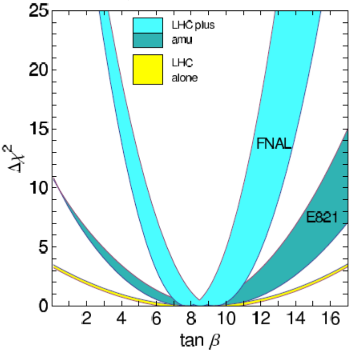
Anomalous magnetic moment:

$$a_\mu \, (\mathsf{SUSY}) \approx \mathsf{sgn}(\mu) \times \left(130 \times 10^{-11}\right) \times \tan\beta \times \left(\frac{100 \, \mathsf{GeV}}{\Lambda}\right)^2$$

Having superpartner masses around a few hundred GeV would explain the anomalous magnetic moment!



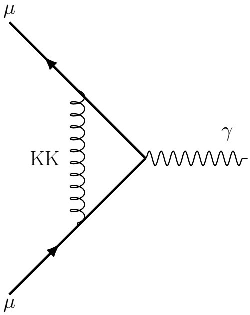
$m_{\tilde{\mu},\tilde{\nu}}$ arbitrary
all data
$m_{\tilde{\mu}_{1,2}}, m_{\tilde{\nu}_\mu} > 1$ TeV
full result
improved one–loop
$m_{\tilde{\mu},\tilde{\nu}} > 1$ TeV
$a_\mu^{SUSY}[10^{-10}]$
M_{LOSP} [GeV]



LHC plus
amu
LHC
alone
FNAL
E821
$\Delta\chi^2$
$\tan\beta$

Extra dimensions

Unavoidable in string theory, but physics effects are *hugely model dependent*! Explicit calculations indicate *small* contributions to a_μ.



Ex.: 5D Randall-Sundrum model (no bells or whistles)

$$a_\mu(\mathsf{RS}) \approx \left(8.8 \times 10^{-11}\right) \times \left(\frac{1\ \mathsf{TeV}}{T}\right) , \quad M_{\mathsf{KK}} \approx 2.5T$$

Unlikely to source the anomalous magnetic moment, despite what some people claim.

Axions

Ubiquitous in string theory, via compactification of higher-form fields.

Example: 5d spacetime with topology $\mathbb{R}^4 \times S^1$. A $U(1)$ gauge field $\mathcal{A}_M(x, x^4)$ will decompose as:

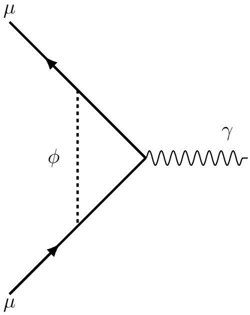
$$A_\mu(x) = \oint_0^R dx^4\, \mathcal{A}_\mu(x, x^4)$$
$$\phi(x) = \oint_0^R dx^4\, \mathcal{A}_4(x, x^4)$$

Original $U(1)$ gauge symmetry is inherited by 4d fields; large gauge transformations $\implies$ shift symmetry $\phi \to \phi + 2\pi R$.

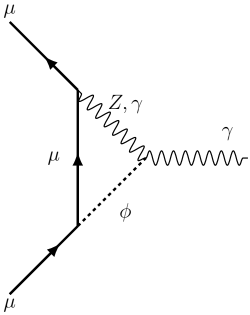
Well-motivated phenomenologically; could explain the strong-CP problem (Peccei-Quinn mechanism) or play the role of an inflaton.

Axions

Scalar Yukawa interactions $\mathcal{L} \supset y\bar{\psi}_L\psi_R\phi$ lead to CP violation!



$a_\mu < 0$



$a_\mu > 0$

Requires unnaturally large coupling $\mathcal{L} \supset \frac{\lambda}{\Lambda}\phi F_{\mu\nu}\widetilde{F}^{\mu\nu}$.

Hard to make work with existing data in simple models, but multitude of axions in string theory may help combat this.

Dark Sectors

String compactifications are hard; extra stuff shows up!

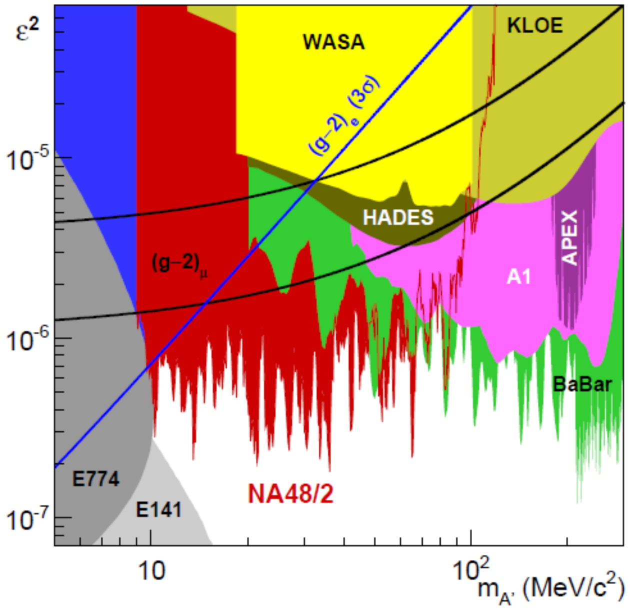
$$G = SU(3) \times SU(2) \times U(1) \times (\text{tons of extra gauge groups})$$

Extra groups not visible, must interact weakly ("dark").

Easiest to study: dark photon A', simple kinetic mixing with SM photon. Couples to SM leptons with strength εe, with $\varepsilon \ll 1$.

$$a_\mu(A') = \frac{\alpha}{2\pi}\varepsilon^2 F\left(\frac{m_{A'}}{m_\mu}\right) , \quad F(x) = \int_0^1 \frac{2z(1-z)^2}{(1-z^2)+x^2 z} dz$$

Requires $\varepsilon^2 \sim 4 \times 10^{-6}$ and 10 MeV $\lesssim m_{A'} \lesssim 100$ MeV.



ε²
10⁻⁵
10⁻⁶
10⁻⁷
10
10²
m_A' (MeV/c²)
WASA
KLOE
(g–2)_e (3σ)
HADES
(g–2)_μ
APEX
A1
BaBar
E774
E141
NA48/2

Conclusions and Optimism

Main experimental takeaways

$g - 2$ is primed to yield new, complementary constraints on supersymmetry!

Split SUSY with extra ingredients? Great!

Less so on other aspects of string theory (extra dimensions, axions, and dark sectors)

But, heavily model-dependent $\implies$ future targets for string model builders!

Quantum gravity elusive, but quantifying these hurdles and discussion with experimentalists is crucial.

QUANTUM GRAVITY AND THE ELECTRON AND MUON ANOMALOUS MAGNETIC MOMENTS

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Quantum gravity corrections at the one-loop level to the electron and muon (g−2)/2 are calculated and found to be finite.

In spite of all the efforts which have been made in quantum gravity, one still does not know any set of elementary particles which, when coupled to the quantized gravitational field, would lead to a renormalizable theory [1]. Since quantum electrodynamics describes electrons and photons so remarkably well, we felt there was perhaps a chance that the inclusion of quantum gravity effects would not lead immediately to meaningless results. So, we chose to calculate the corrections due to graviton exchange to the electron and muon anomaly at the one-loop level, and found a finite result.

For the muon, $a_\mu^G \approx 4.2 \times 10^{-41}$, and for the electron, $a_e^G \approx 1.0 \times 10^{-45}$.

Of course, these numbers are much too small to be of experimental relevance, but the fact that the divergences of the individual diagrams cancel seems rather remarkable.